THE EXAMPLE OF USING THE LIMIT COMPARISON TEST PRESENTED IN CLASS.

<u>EXAMPLE</u>: Is $\sum_{n=1}^{\infty} \frac{4n+3}{\sqrt{n^5+n^3+1}} = \sum_{n=1}^{\infty} a_n$ C or D?

<u>Sol'n</u>: For the comparison series $\sum_{n=1}^{\infty} b_n$, what should b_n be?

$$b_n = \frac{n}{\sqrt{n^5}} = \frac{n}{n^{5/2}} = \frac{1}{n^{3/2}} \quad \text{and} \quad \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

THE FIRST JUSIFICATION (REQUIRED)

"$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is a p-series with $p = \frac{3}{2} > 1$, so $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is convergent."

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{4n+3}{\sqrt{n^5+n^3+1}}\right)\left(\frac{n^{3/2}}{1}\right)$$

$$= \lim_{n\to\infty} \left(\frac{n\,(4+3/n)}{\sqrt{n^5}\,\sqrt{1+1/n^2+1/n^5}}\right)\left(\frac{n^{3/2}}{1}\right) = \lim_{n\to\infty} \underbrace{\frac{n^{5/2}}{n^{5/2}}}_{\text{This equals } 1.} \left(\frac{(4+3/n)}{\sqrt{1+\frac{1}{n^2}+\frac{1}{n^5}}}\right) = 4.$$

THE SECOND JUSTIFICATION (REQ'D).

"Because $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is convergent,

and $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(\frac{4n+3}{\sqrt{n^5+n^3+1}}\right)}{\left(\frac{1}{n^{3/2}}\right)} = 4$ and 4 is a

finite non-zero number, the series $\sum_{n=1}^{\infty} \frac{4n+3}{\sqrt{n^5+n^3+1}}$ is

convergent by the Limit Comparison Test."